Economical map-based turbulence models

Developments and perspectives for the numerical analysis of electrostatic precipitation

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1. Introduction

In an age of high-performance applications demanding close to optimal operation, vast complex electrostatic precipitators (ESPs) are simulated using turbulence models which allow predictions of high accuracy, but which are becoming highly or overly sophisticated in an attempt to cover large operational ranges. This is the case of models which do not resolve relevant physical scales below a threshold scale, but instead model physical phenomena on those non-resolved scales, i.e., simulation approaches based on the Reynolds-Averaged Navier-Stokes (RANS) equations or Large Eddy Simulations (LES) [1]. In contrast to these turbulence models, if the fundamental mode of action of the relevant physical phenomena and their interactions is well understood, even simple and reduced order models can be applicable. Recall that transport phenomena are usually represented by nondimensional numbers. This involves, first and foremost, the Reynolds number of the flow (Re), which can span more than three orders of magnitude ranging from $O(10^3)$ in laboratory prototypes up to O(10⁶) in large-scale industrial ESPs. The Schmidt number (Sc) is also relevant, varying from O(1) to O(10⁵), when different gas species and solid particles present on exhaust gases entering the ESP are considered. Further attention must be paid to the electrohydrodynamic (EHD) body force magnitude, characterized by an EHD number (N_{EHD}) or a Masuda number (Md) [2], and last but not least, the ratio between the flow time scale and the particle phase relaxation time scale, i.e., the Stokes number (St), which could also be rewritten as a mobility ratio (M) for charged particles. Available literature and understanding of this vast parameter space is, in the best case, scarce. Consequently, the reliability on the highly sophisticated and potentially overfitted RANS and LES subgrid-scale model parametrizations is an issue of concern. As an alternative, our group has dedicated significant time and effort to the development of a family of seemingly simplistic stochastic turbulence models, which nonetheless often surprise in performance and also offer great opportunities in understanding fundamental physical laws. Here, we discuss the rationale of these models. Additionally, we present study cases for particle-free EHD flows considering simplified ESPlike configurations, and particle-laden turbulent shear flows, relevant for aerosol- or particleladen ESP flows. Interesting insights concerning effects of the EHD body force on drag are obtained [3, 4], as well as relevant findings concerning turbulence modification by particles [5]. Finally, we discuss perspectives regarding ongoing and future work.

2. Model rationale

A key issue for the simulation of turbulent flows is the representation of the kinetic energy cascade phenomenology. On the assumption that small scale processes yield the collective behavior of continuum flow, the map-based modeling rationale in the One-Dimensional Turbulence (ODT) focuses on a simplified representation of scalar advection by small scale turbulence, e.g., the modeling of the nonlinear advection terms in the governing equations. Essentially, this is a Lagrangian mapping of fluid parcels [6, 7].



Fig. 1: ODT Model representation of the effects of a turbulent eddy (right) on an otherwise axisymmetric density field with uniform gradient in wall normal direction (left).

By modeling the nonlinearities, the computational cost reduces significantly. For flows with statistically dominant 1-D flow gradients, which are the majority of internal flows with flow forcing and even boundary layer flows, once the nonlinearity (inherently 3-D) is modeled, the flow can be simulated as a Direct Numerical Simulation (DNS) in a reduced dimensional setting, i.e., a 1-D domain. The suggested mappings are triplet maps of 1-D scalar profiles $\varphi(y)$, in some range of extent [y₀, y₀ + I], see Fig. 1. The mappings represent the effects of turbulent eddies in the statistical ensemble limit, and therefore, I is a turbulent eddy length scale, and y₀ is the corresponding eddy position. Mapping of the velocity components as scalars implies that every model-represented turbulent eddy has also an associated eddy turnover time. Following this, a stochastic process can be formulated to sample mappings based on their individually calculated rate (flow state).

3. Model application on relevant ESP-like flows

Recently, the authors proposed an implementation of ODT in canonical channels and pipe flows with the presence of inhomogeneous EHD body forces. This was done by the incorporation of one or various electrodes in the flow responsible for an electrical discharge, and consequently inhomogeneous electric fields, see Fig. 2. In [3], ODT was applied in a channel with a periodic array of electrodes located at the channel centerline. This configuration resembles a wire-plate ESP. The goal was the validation of the model in the presence of the inhomogeneous 2-D electric fields caused by the electrode array, verifying mainly the effects of the electric field on drag modification. As in the DNS, the fixed pressure gradient forced flow evidenced a drag reduction effect as a consequence of larger electrical body forces (larger N_{EHD}). In [4, 8], ODT was applied in a developing pipe flow with a concentric electrode, resembling a wire-tube ESP. The wire-tube simulation results shed light on the different contributions to the pressure drop between the inlet and the outlet of the device, which is generally measured in experimental tests, and which sometimes is wrongly associated to the friction factor or friction losses. In fact, [4, 8] shows that the measured pressure drop $d\bar{p}/dz$, defined as the Darcy friction factor $f_D = (-d\bar{p}/dz)^*2D/(\rho_b U_b^2)$ in the experimental work, utilizing the density ρ_b and velocity U_b of the bulk flow, behaves in a different way when compared to the wall shear stress τ_w -based skin friction $C_f = 2\tau_w/(\rho_b U_b^2)$, since the Coulomb force, the kinetic energy gradient and τ_w contribute all to $d\bar{p}/dz$, see Fig. 3.



Fig. 2: ODT simulation results showing contours of electrostatic potential superimposed on the kinetic energy of the mean flow in a wire-plate ESP-like configuration with a periodic electrode array (left), and a wire-tube ESP-like flow (right). Bottom-to-top gas flow.



Fig. 3: Wire-tube ESP flow simulation data showing the difference between C_f (left) and $f_D/4$ (right). Refer to [4, 8] for details. $C_f = f_D/4$ in traditional pipe-flow.

Finally, we comment on a novel implementation of ODT for particle-laden flows [5], which utilizes spherical point-particles in the 1-D domain following a modified drag law. The particle-eddy interaction model [5] allows both one-way and two-way couplings between the

flow field and the Lagrangian particles. This is required for the evaluation of turbulence modulation effects, see Fig. 4.



Fig. 4: Turbulence Kinetic Energy (TKE) k_{pl} (relative to particle-free flow TKE, k_{pl}) as a function of the particle mass loading for homogeneous turbulent shear flow (left) [5]. Lagrangian particle trajectories in a turbulent channel flow (St ~ 7) are also shown (right). Bottom-to-top flow.

4. Perspectives and Conclusions

We have demonstrated that the application of stochastic map-based turbulence models for the simulation of industrial ESPs is feasible, and several insights, including fundamental physical ones, can be derived from their use. The application of these models is advantageous, given that small scale processes are directly represented, and no further parameter-fitting is required for new physics. The only model parameter calibration, in that sense, is that required to match the eddy event implementation rate with that yielding consistent statistical moments of the flow. Future work will extend the current model formulation to include the effects of two-way coupled electric fields by the charging kinetics of an Eulerian particle-density field. The effects of the Lagrangian particle field (one-way and two-way) on the current particle-free EHD ODT solver will also be evaluated in the future.

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